Tutorial "Performance Evaluation Techniques" Third Problem Sheet

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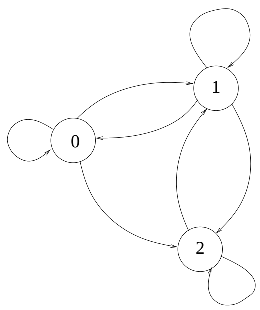
Problem 1:
In the tutorial we discussed one modeling problem involving a TH-DTMC:



with transition matrix:

$$\mathbf{P} = \begin{pmatrix} 1-p & p(1-p) & p^2 \\ 1-p & p(1-p) & p^2 \\ 0 & 1-p & p \end{pmatrix}$$

where $p \in (0,1)$ is a parameter. Show that:

- The TH-DTMC is irreducible and aperiodic
- Find the steady-state vector $\pi = (\pi^{(0)}, \pi^{(1)}, \pi^{(2)})$ (this vector is guaranteed to exist, since all finite, irreducible and aperiodic TH-DTMCs are also positive recurrent).

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Problem 2:
(Modeling Problem) Consider a computer system with two identical processors working in parallel. Time is slotted. The system works according to the following rules:

- In each time slot at most one new task arrives, which happens with probability $\alpha \in (0,1)$ per slot. Task arrivals are independent.
- If one processor is available, the task is immediately started at this processor.
- If two processors are available, the task is immediately started on processor 1.
- If both processors are busy, the task is lost.
- A single processor ends a task in a slot with probability β, the tasks are independent. (Hence, the event that both processors end their tasks is given by β^2). Ended tasks leave the system.
- If a new task arrives in a slot where at least one processor ends a task, the task will be served.

Develop a TH-DTMC model for this system. Let the state variable X_n denote the number of busy servers during time slot n.

- Draw a diagram showing the possible state transitions.
- Find the state transition probabilities and give the state transition matrix $\mathbf{P}$.
- Find the steady-state vector.
- For $\alpha = \beta = 0.01$ compute the steady state vector and the mean utilization:

$$\sum_{i=0}^{2} i \cdot \pi^{(i)}$$

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Bonus Problem 3:
In another problem discussed in the tutorial we developed the following matrix of a TH-DTMC:

$$\mathbf{P} = \begin{pmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ b(0;1,p) & b(1;1,p) & 0 & 0 & \dots & 0 \\ b(0;2,p) & b(1;2,p) & b(2;2,p) & 0 & \dots & 0 \\ \dots & & & & & \\ b(0;N,p) & b(1;N,p) & b(2;N,p) & b(3;N,p) & \dots & b(N;N,p) \end{pmatrix}$$

where $p \in (0,1)$ is a parameter, $b(k;n,p) = \binom{n}{k} p^k (1-p)^{n-k}$ is the distribution function of the binomial distribution and $\mathbf{P}$ is an $(N+1) \times (N+1)$ matrix. Use $p = 0.3$, $N = 10$ and the initial state vector is

$$\pi_0 = (0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1)$$

Print $\pi_k = \pi_{k-1} \cdot \mathbf{P} = \pi_0 \cdot \mathbf{P}^k$ for $k \in \{1, 2, 5, 8, 10\}$. Write a program/script using a suitable mathematics package (`maxima/xmaxima`, GNU `octave`, `scilab`) or in your favorite programming language.

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