Tutorial "Performance Evaluation Techniques" Fourth Problem Sheet

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Problem 1 (Required!):
Install the OMNet++ discrete event simulation package in the latest version on your computer and run the test simulations. You can find the software at:
`http://whale.hit.bme.hu/omnetpp/`.
Read the Manual. Set up a M/M/2 simulation. Vary the load ρ as $\rho \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8\}$ (assume a service rate μ of one customer per second and vary λ accordingly). For each ρ create 10000 customers (the initial number of customers in the system is zero).

- determine the mean system response time for all the customers (sample mean)
- look every 0.1 seconds at the system, observe the number of customers in the system at each sampling point and count how often exactly k customers are found ($k \geq 0$). Plot the relative frequencies in a histogram.

Compare your simulation results with analytical results.
Submit your code (C++), your simulation results and the correct analytical results.

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Problem 2:
Use the Pollaczek-Khintchine mean value formula to show that a M/M/1 system has twice the expected number of customers in the system as the M/D/1 system as $\rho \rightarrow 1$ ($\rho < 1$)

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Problem 3:
Consider the M/M/N/N loss system ($N \geq 1$) with arrival rate λ and μ being the rate of a single server.

- (0.5) Draw the state diagram
- (0.5) Give the generator matrix $\mathbf{Q}$
- (1.5) Find the steady-state vector $\pi = (\pi_0, \pi_1, \ldots, \pi_N)$
- (0.5) Using this, show that with $\rho = \frac{\lambda}{\mu}$:

  $$\Pr[\text{Customer loss}] = \pi_N = \frac{\rho^N}{N!} \frac{1}{\sum_{k=0}^{N} \frac{\rho^k}{k!}}$$

  (this is the *Erlang loss formula*)
- (1) Assume $\rho = \frac{\lambda}{\mu} = 5$ and evaluate $\Pr[\text{Customer loss}]$ for $N = 1, 2, 3, \ldots, 20$.
- (1) How might a telephone company use this formula?

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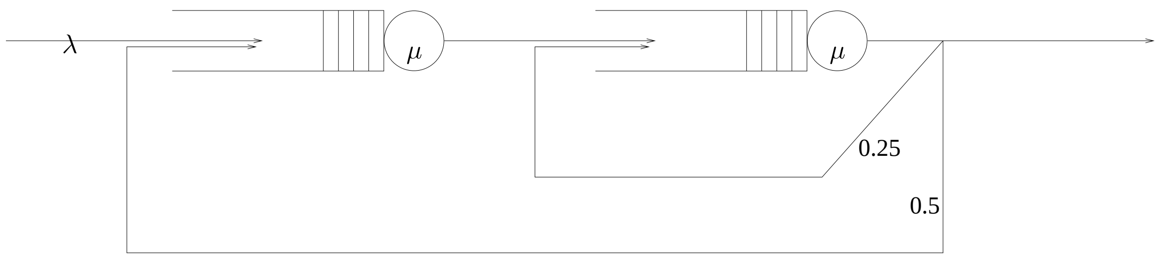
Problem 4:
Consider the fully Markovian queueing network shown in this figure:



- Find a stability condition for this system.
- Find the mean time for a customer to proceed through the system.

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